Regular Expressions
and
Finite State Automata

Introduction

- Regular expressions are equivalent to Finite State Automata in recognizing regular languages, the first step in the Chomsky hierarchy of formal languages
- The term regular expressions is also used to mean the extended set of string matching expressions used in many modern languages
  - Some people use the term regexp to distinguish this use
- Some parts of regexps are just syntactic extensions of regular expressions and can be implemented as a regular expression – other parts are significant extensions of the power of the language and are not equivalent to finite automata

Concepts and Notations

- Set: An unordered collection of unique elements

  S_1 = { a, b, c } S_2 = { 0, 1, ..., 19 }   *empty set:* $\varnothing$

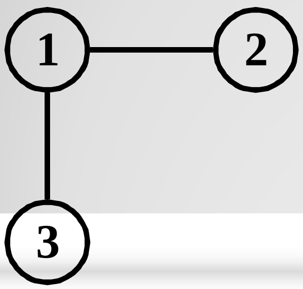
  membership: $x \in S$ *union:* $S_1 \cup S_2$ = { a, b, c, 0, 1, ..., 19 }

  universe of discourse: U *subset:* $S_1 \subset U$

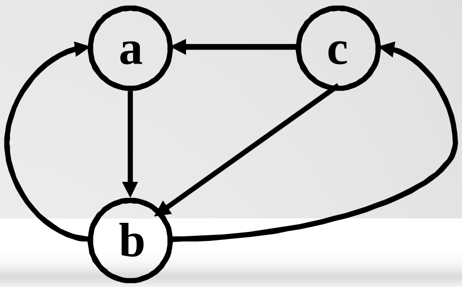
  complement: if U = { a, b, ..., z }, then S_1' = { d, e, ..., z } = $U - S_1$
- Alphabet: A finite set of symbols
  - Examples:
    - Character sets: ASCII, ISO-8859-1, Unicode
    - S_1 = { a, b } S_2 = { Spring, Summer, Autumn, Winter }
- String: A sequence of zero or more symbols from an alphabet
  - The empty string: e

Concepts and Notations

- Language: A set of strings over an alphabet
  - Also known as a formal language; may not bear any resemblance to a natural language, but could model a subset of one.
  - The language comprising **all** strings over an alphabet ∑ is written as: ∑*
- Graph: A set of nodes (or vertices), some or all of which may be connected by edges.
  - An example:



– A directed graph example:



Regular Expressions

- A regular expression defines a regular language over an alphabet ∑:
  - ∅ is a regular language: //
  - Any symbol from ∑ is a regular language:

    ∑ = { a, b, c}   /a/   /b/   /c/
  - Two concatenated regular languages is a regular language:

    ∑ = { a, b, c}   /ab/   /bc/   /ca/

Regular Expressions

- Regular language (continued):
  - The union (or disjunction) of two regular languages is a regular language:

    $\Sigma = \{ a, b, c\}$ /ab|bc/ /ca|bb/
  - The Kleene closure (denoted by the Kleene star: *) of a regular language is a regular language:

    $\Sigma = \{ a, b, c\}$ /a*/ /(ab|ca)*/
  - Parentheses group a sub-language to override operator precedence (and, we'll see later, for "memory").

Finite Automata

- Finite State Automaton

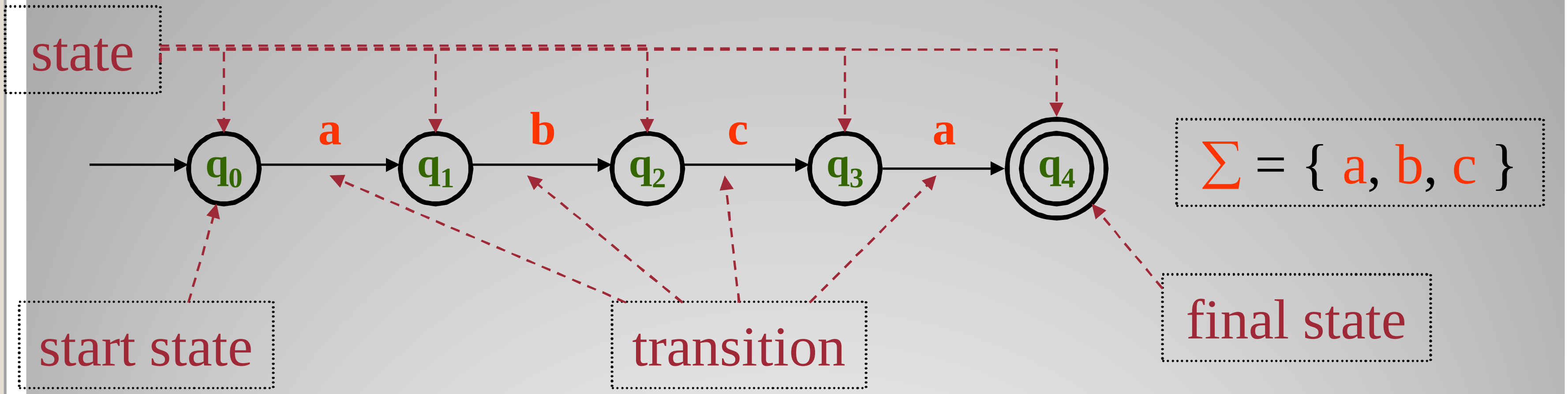
  a.k.a. Finite Automaton, Finite State Machine, FSA or FSM

  - An abstract machine which can be used to implement regular expressions (etc.).
  - Has a finite number of states, and a finite amount of memory (i.e., the current state).
  - Can be represented by directed graphs or transition tables

Finite-state Automata (1/23)

- Representation
  - An FSA may be represented as a directed graph; each node (or vertex) represents a state, and the edges (or arcs) connecting the nodes represent transitions.
  - Each state is labelled.
  - Each transition is labelled with a symbol from the alphabet over which the regular language represented by the FSA is defined, or with e, the empty string.
  - Among the FSA's states, there is a start state and at least one final state (or accepting state).

Finite-state Automata (2/23)



- Representation (continued)
  - An FSA may also be represented with a state-transition table. The table for the above FSA:

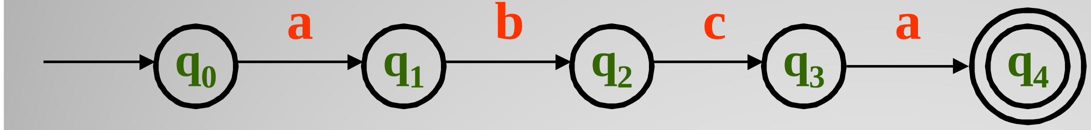
	Input		
State	a	b	c
0	1	∅	∅
1	∅	2	∅
2	∅	∅	3
3	4	∅	∅
4	∅	∅	∅

Finite-state Automata (3/23)

- Given an input string, an FSA will either accept or reject the input.
  - If the FSA is in a final (or accepting) state after all input symbols have been consumed, then the string is accepted (or recognized).
  - Otherwise (including the case in which an input symbol cannot be consumed), the string is rejected.

Finite-state Automata (3/23)

$\Sigma = \{ a, b, c \}$



	Input		
State	a	b	c
0	1	∅	∅
1	∅	2	∅
2	∅	∅	3
3	4	∅	∅
4	∅	∅	∅

IS_1:

a	b	c	a

IS_2:

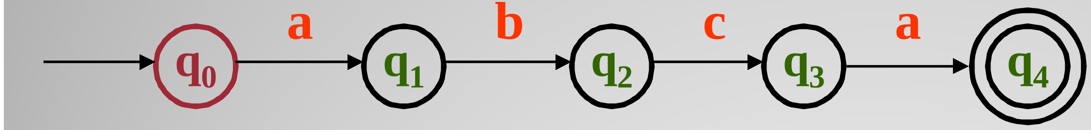
c	c	b	a

IS_3:

a	b	c	a	c

Finite-state Automata (4/23)

$\sum = \{ a, b, c \}$



State	Input		
	a	b	c
0	1	∅	∅
1	∅	2	∅
2	∅	∅	3
3	4	∅	∅
4	∅	∅	∅

IS$_1$:

a	b	c	a

IS$_2$:

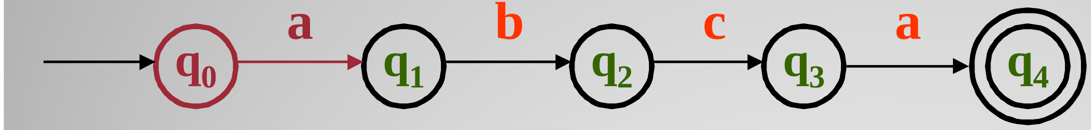
c	c	b	a

IS$_3$:

a	b	c	a	c

Finite-state Automata (5/23)

$\sum = \{ a, b, c \}$

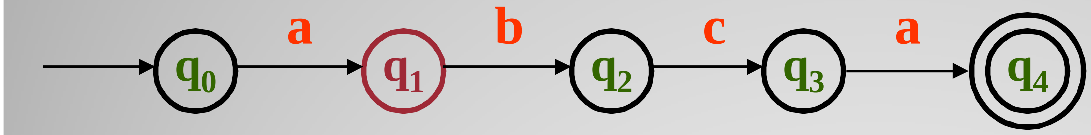


IS1:	**a**	b	c	a	
IS2:	c	c	b	a	
IS3:	a	b	c	a	c

	Input		
State	**a**	**b**	**c**
0	**1**	∅	∅
1	∅	2	∅
2	∅	∅	3
3	4	∅	∅
4	∅	∅	∅

Finite-state Automata (6/23)

$\sum = \{ a, b, c \}$



IS$_1$:

a	b	c	a

IS$_2$:

c	c	b	a

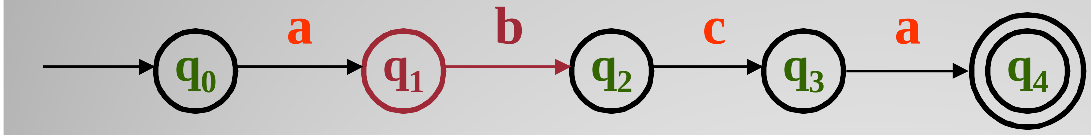
IS$_3$:

a	b	c	a	c

	Input		
State	a	b	c
0	1	∅	∅
1	∅	2	∅
2	∅	∅	3
3	4	∅	∅
4	∅	∅	∅

Finite-state Automata (7/23)

$\sum = \{ a, b, c \}$



	Input		
State	a	b	c
0	1	∅	∅
1	∅	2	∅
2	∅	∅	3
3	4	∅	∅
4	∅	∅	∅

IS_1:

a	b	c	a

IS_2:

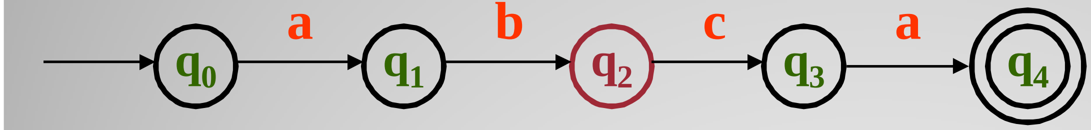
c	c	b	a

IS_3:

a	b	c	a	c

Finite-state Automata (8/23)

$\Sigma = \{ a, b, c \}$

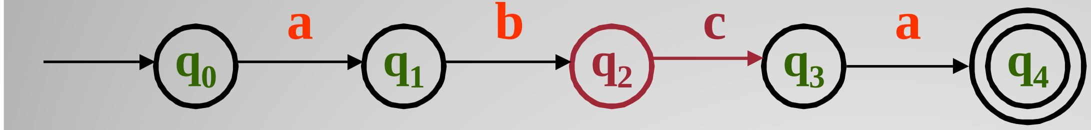


IS$_1$:	a	b	c	a	
IS$_2$:	c	c	b	a	
IS$_3$:	a	b	c	a	c

	Input		
State	a	b	c
0	1	∅	∅
1	∅	2	∅
2	∅	∅	3
3	4	∅	∅
4	∅	∅	∅

Finite-state Automata (9/23)

$\sum = \{ a, b, c \}$



IS1:

a	b	c	a

IS2:

c	c	b	a

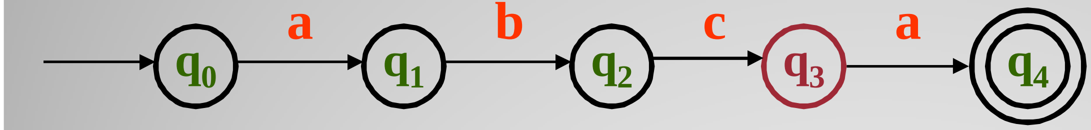
IS3:

a	b	c	a	c

	Input		
State	a	b	c
0	1	∅	∅
1	∅	2	∅
2	∅	∅	3
3	4	∅	∅
4	∅	∅	∅

Finite-state Automata (10/23)

$\sum = \{ a, b, c \}$

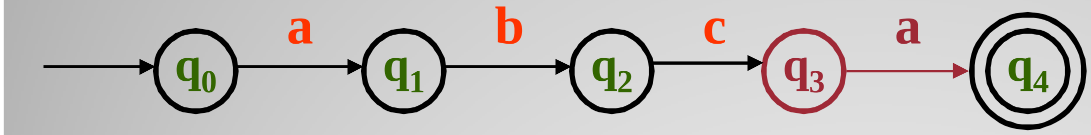


IS$_1$:	a	b	c	a	
IS$_2$:	c	c	b	a	
IS$_3$:	a	b	c	a	c

	Input		
State	a	b	c
0	1	∅	∅
1	∅	2	∅
2	∅	∅	3
3	4	∅	∅
4	∅	∅	∅

Finite-state Automata (11/23)

$\sum = \{ a, b, c \}$

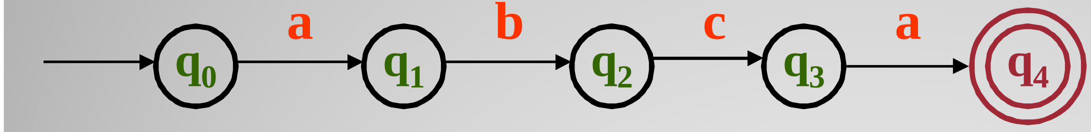


IS$_1$:	a	b	c	a
IS$_2$:	c	c	b	a
IS$_3$:	a	b	c	a	c

State	Input a	 b	 c
0	1	∅	∅
1	∅	2	∅
2	∅	∅	3
3	4	∅	∅
4	∅	∅	∅

Finite-state Automata (12/23)

$\sum = \{ a, b, c \}$



IS$_1$:

a	b	c	a

IS$_2$:

c	c	b	a

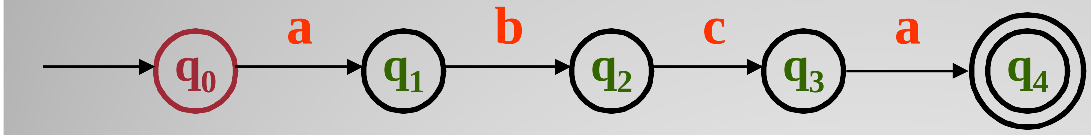
IS$_3$:

a	b	c	a	c

	Input		
State	a	b	c
0	1	∅	∅
1	∅	2	∅
2	∅	∅	3
3	4	∅	∅
4	∅	∅	∅

Finite-state Automata (13/23)

$\Sigma = \{ a, b, c \}$



		Input	
State	a	b	c
0	1	∅	∅
1	∅	2	∅
2	∅	∅	3
3	4	∅	∅
4	∅	∅	∅

IS_1:

a	b	c	a

IS_2:

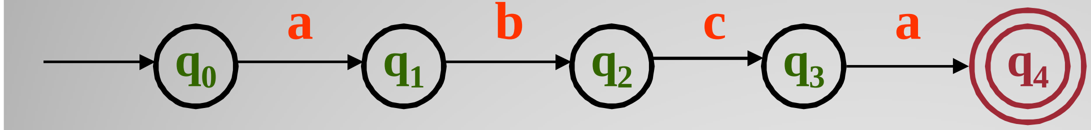
c	c	b	a

IS_3:

a	b	c	a	c

Finite-state Automata (14/23)

$\sum = \{ a, b, c \}$



IS₁:				
	a	b	c	a

IS₂:				
	c	c	b	a

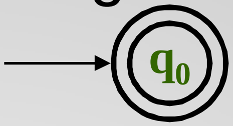
IS₃:					
	a	b	c	a	c

	Input		
State	a	b	c
0	1	∅	∅
1	∅	2	∅
2	∅	∅	3
3	4	∅	∅
4	∅	∅	∅

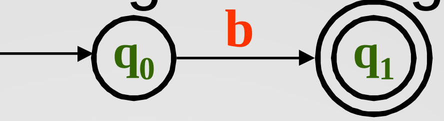
Finite-state Automata (22/23)

- An FSA defines a regular language over an alphabet Σ:
  - $\varnothing$ is a regular language:
  - Any symbol from Σ is a regular language:

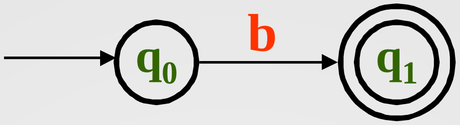
    Σ = { a, b, c}

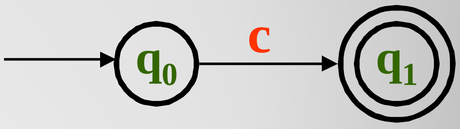


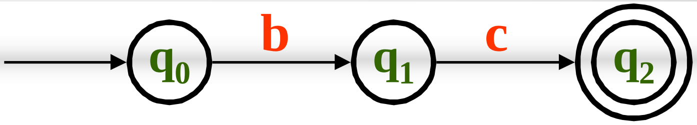
  - Two concatenated regular languages is a regular language:



    Σ = { a, b, c}



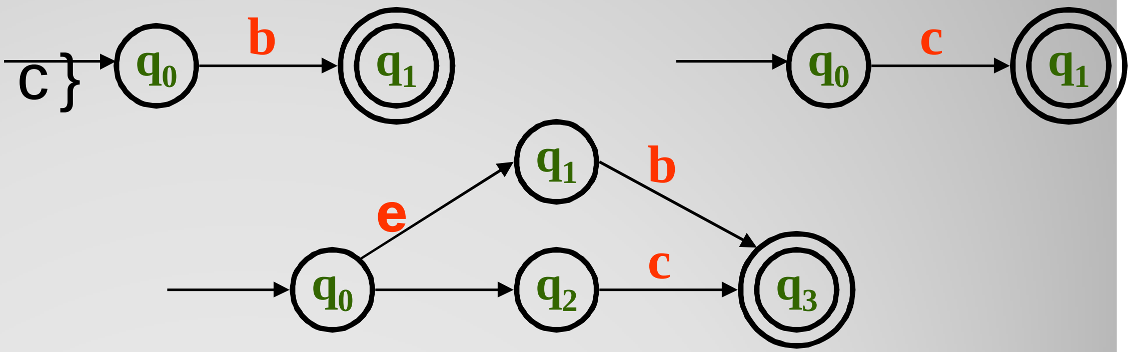




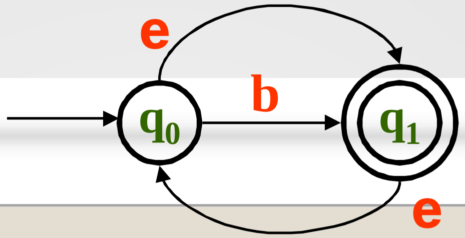
Finite-state Automata (23/23)

- regular language (continued):
  - The union (or disjunction) of two regular languages is a regular language:

$\sum$ = { a, b, c}



  - The Kleene closure (denoted by the Kleene star: *) of a regular language is a regular language:



$\sum$ = { a, b, c}

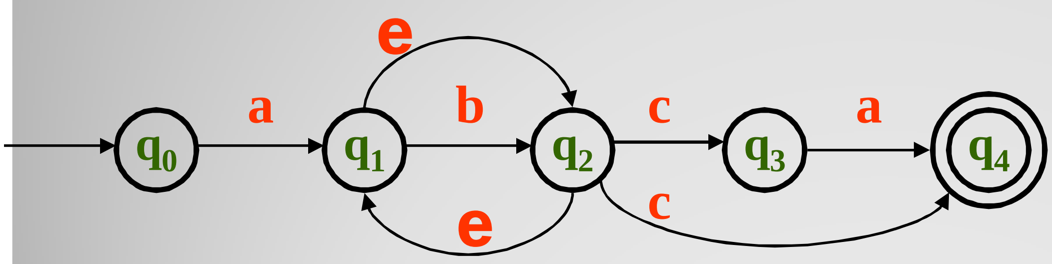
Finite-state Automata (15/23)

- Determinism
  - An FSA may be either deterministic (DFSA or DFA) or non-deterministic (NFSA or NFA).
    - An FSA is deterministic if its behavior during recognition is fully determined by the state it is in and the symbol to be consumed.
      - I.e., given an input string, only one path may be taken through the FSA.
    - Conversely, an FSA is non-deterministic if, given an input string, more than one path may be taken through the FSA.
      - One type of non-determinism is e-transitions, i.e. transitions which consume the empty string (no symbols).

Finite-state Automata (16/23)

- An example NFA:

$\Sigma = \{ a, b, c \}$



	Input			
State	a	b	c	e
0	1	∅	∅	∅
1	∅	2	∅	2
2	∅	∅	3,4	1
3	4	∅	∅	∅
4	∅	∅	∅	∅

– The above NFA is equivalent to the regular expression /ab*ca?/.

Finite-state Automata (17/23)

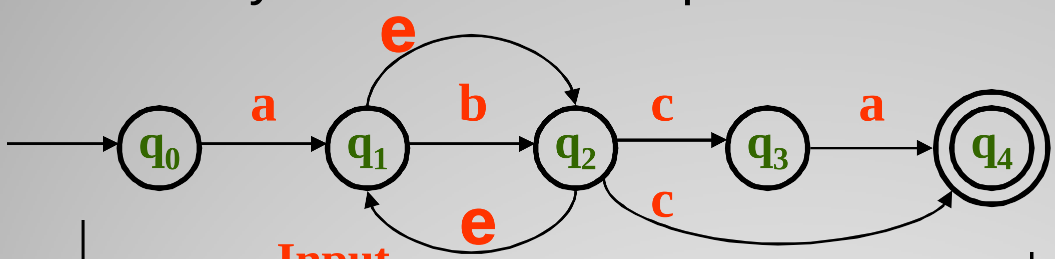
- String recognition with an NFA:
  - Backup (or backtracking): remember choice points and revisit choices upon failure
  - Look-ahead: choose path based on foreknowlege about the input string and available paths
  - Parallelism: examine all choices simultaneously

Finite-state Automata (18/23)

- Recognition as search
  - Recognition can be viewed as selection of the correct path from all possible paths through an NFA (this set of paths is called the state-space)
  - Search strategy can affect efficiency: in what order should the paths be searched?
    - Depth-first (LIFO [last in, first out]; stack)
    - Breadth-first (FIFO [first in, first out]; queue)
    - Depth-first uses memory more efficiently, but may enter into an infinite loop under some circumstances

Finite-state Automata (19/23)

- Conversion of NFAs to DFAs
  - Every NFA can be expressed as a DFA.



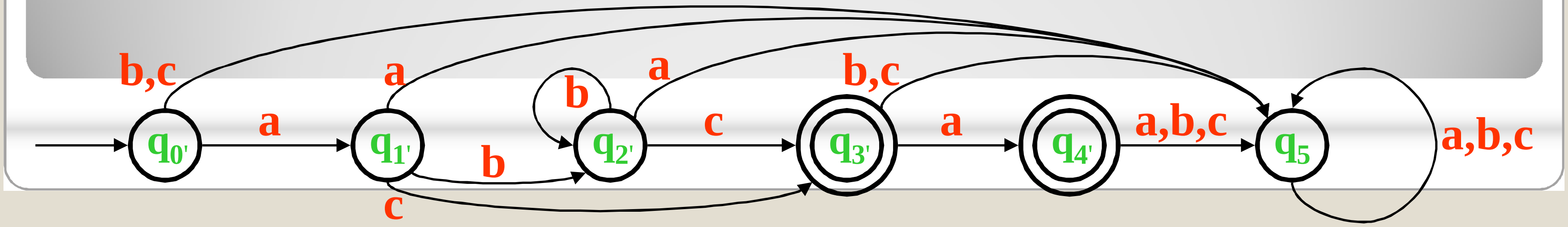
$\sum = \{ a, b, c \}$

/ab*ca?/

	Input			
State	**a**	**b**	**c**	**e**
0	1	∅	∅	∅
1	∅	2	∅	2
2	∅	∅	3,4	1
3	4	∅	∅	∅
4F	∅	∅	∅	∅

Subset construction ⟹

		Input		
New State	State	**a**	**b**	**c**
0'	0	1	∅	∅
1'	1	∅	2	{3,4}
2'	2	∅	2	{3,4}
3'F	{3,4}F	4	∅	∅
4'F	4F	∅	∅	∅
5	∅	∅	∅	∅

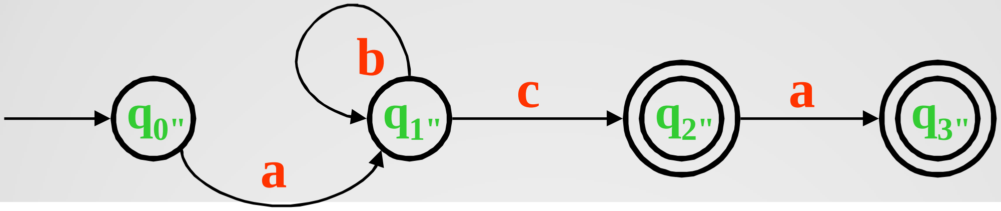


Finite-state Automata (20/23)

- DFA minimization
  - Every regular language has a unique minimum-state DFA.
  - The basic idea: two states s and t are *equivalent* if for every string w, the transitions $T(s, w)$ and $T(t, w)$ are both either final or non-final.
  - An algorithm:
    - Begin by enumerating all possible pairs of both final or both non-final states, then iteratively removing those pairs the transition pair for which (for any symbol) are either not equal or are not on the list. The list is complete when an iteration does not remove any pairs from the list.
    - The minimum set of states is the partition resulting from the unions of the remaining members of the list, along with any original states not on the list.

Finite-state Automata (21/23)

- The minimum-state DFA for the DFA converted from the NFA for /ab*ca?/, without the “failure” state (labeled “5”), and with the states relabeled to the set $Q = \{ q_{0''}, q_{1''}, q_{2''}, q_{3''} \}$:



Finite Automata with Output

- Finite Automata may also have an output alphabet and an action at every state that may output an item from the alphabet
- Useful for lexical analyzers
  - As the FSA recognizes a token, it outputs the characters
  - When the FSA reaches a final state and the token is complete, the lexical analyzer can use
    - Token value – output so far
    - Token type – label of the output state

RegExps

- The extended use of regular expressions is in many modern languages:
  - Perl, php, Java, python, …
- Can use regexps to specify the rules for any set of possible strings you want to match
  - Sentences, e-mail addresses, ads, dialogs, etc
- ``Does this string match the pattern?'', or ``Is there a match for the pattern anywhere in this string?''
- Can also define operations to do something with the matched string, such as extract the text or substitute for it
- Regular expression patterns are compiled into a executable code within the language

Regular Expressions

- Regexp syntax is a superset of the notation required to express a regular language.
  - Some examples and shortcuts:
    1. `/[abc]/ = /a|b|c/` Character class; disjunction
    2. `/[b-e]/ = /b|c|d|e/` Range in a character class
    3. `/[\012\015]/ = /\n|\r/` Octal characters; special escapes
    4. `/./ = /[\x00-\xFF]/` Wildcard; hexadecimal characters
    5. `/[^b-e]/ = /[\x00-af-\xFF]/` Complement of character class
    6. `/a*/   /[af]*/   /(abc)*/` Kleene star: zero or more
    7. `/a?/ = /a|/   /(ab|ca)?/` Zero or one
    8. `/a+/   /([a-zA-Z]1|ca)+/` Kleene plus: one or more
    9. `/a{8}/   /b{1,2}/   /c{3,}/` Counters: exact repeat quantification

Regular Expressions

- Anchors
  - Constrain the position(s) at which a pattern may match
  - Think of them as “extra” alphabet symbols, though they actually consume e (the zero-length string):
  - `/^a/` Pattern must match at beginning of string
  - `/a$/` Pattern must match at end of string
  - `/\bword23\b/` “Word” boundary: `/[a-zA-Z0-9_][^a-zA-Z0-9_]/` or `/[^a-zA-Z0-9_][a-zA-Z0-9_]/`
  - `/\B23\B/` “Word” **non**-boundary

Regular Expressions

- Escapes
  - A backslash “\” placed before a character is said to “escape” (or “quote”) the character. There are six classes of escapes:
    1. **Numeric character representation**: the octal or hexadecimal position in a character set: “\012” = “\xA”
    2. **Meta-characters**: The characters which are syntactically meaningful to regular expressions, and therefore must be escaped in order to represent themselves in the alphabet of the regular expression: “[](){}|^$.?+*\” (note the inclusion of the backslash).
    3. **“Special” escapes** (from the “C” language):

newline: “\n” = “\xA” carriage return: “\r” = “\xD”

tab:“\t” = “\x9” formfeed: “\f” = “\xC”

Regular Expressions

- ## Escapes (continued)
  - Classes of escapes (continued):

4. **Aliases**: shortcuts for commonly used character classes. (Note that the capitalized version of these aliases refer to the complement of the alias's character class):

whitespace:	`“\s”` = `“[ \t\r\n\f\v]”`
digit:	`“\d”` = `“[0-9]”`
word:	`“\w”` = `“[a-zA-Z0-9_]”`
non-whitespace:	`“\S”` = `“[^ \t\r\n\f]”`
non-digit:	`“\D”` = `“[^0-9]”`
non-word:	`“\W”` = `“[^a-zA-Z0-9_]”`

5. **Memory/registers/back-references**: `“\1”`, `“\2”`, etc.
6. **Self-escapes**: any character other than those which have special meaning can be escaped, but the escaping has no effect: the character still represents the regular language of the character itself.

Regular Expressions

- Memory/Registers/Back-references
  - Many regular expression languages include a memory/register/back-reference feature, in which sub-matches may be referred to later in the regular expression, and/or when performing replacement, in the replacement string:
    - Perl: `/(\w+)\s+\1\b/` matches a repeated word
    - Python: `re.sub(”(the\s+)the(\s+|\b)”,”\1”,string)` removes the second of a pair of ‘the’s
  - **Note: finite automata cannot be used to implement the memory feature.**

Regular Expression Examples

Character classes and Kleene symbols

[A-Z] = one capital letter

[0-9] = one numerical digit

[st@!9] = s, t, @, ! or 9

[A-Z] matches G or W or E

does not match GW or FA or h or fun

[A-Z]+ = one or more consecutive capital letters

matches GW or FA or CRASH

[A-Z]? = zero or one capital letter

[A-Z]* = zero, one or more consecutive capital letters

matches on eat or EAT or I

so, [A-Z]ate

matches Gate, Late, Pate, Fate, but not GATE or gate

and [A-Z]+ate

matches: Gate, GRate, HEate, but not Grate or grate or STATE

and [A-Z]*ate

matches: Gate, GRate, and ate, but not STATE, grate or Plate

Regular Expression Examples (cont'd)

[A-Za-z] = any single letter

so [A-Za-z]+

matches on any word composed of only letters,

but will not match on "words": bi-weekly , yes@SU or IBM325

they will match on bi, weekly, yes, SU and IBM

a shortcut for [A-Za-z] is \w, which in Perl also includes _

so (\w)+ will match on Information, ZANY, rattskellar and jeuvbaew

\s will match whitespace

so (\w)+(\s)(\w+) will match real estate or Gen Xers

Regular Expression Examples (cont’d)

Some longer examples:

([A-Z][a-z]+)\s([a-z0-9]+)

matches: Intel c09yt745 but not IBM series5000

[A-Z]\w+\s\w+\s\w+[!]

matches: The dog died!

It also matches that portion of “ he said, “ The dog died! “

[A-Z]\w+\s\w+\s\w+[!]$

matches: The dog died!

But does not match “he said, “ The dog died! “ because the $ indicates end of Line, and there is a quotation mark before the end of the line

(\w+ats?\s)+

parentheses define a pattern as a unit, so the above expression will match:

Fat cats eat Bats that Splat

Regular Expression Examples (cont'd)

To match on part of speech tagged data:

```
(\w+[-]?\w+\|[A-Z]+)     will match on:
      bi-weekly|RB
   camera|NN
   announced|VBD

(\w+\|V[A-Z]+)   will match on:
   ruined|VBD
   singing|VBG
   Plant|VB
   says|VBZ

(\w+\|VB[DN])  will match on:
   coddled|VBN
   Rained|VBD
   But not  changing|VBG
```

Regular Expression Examples (cont’d)

Phrase matching:

a\|DT ([a-z]+\|JJ[SR]?) (\w+\|N[NPS]+)

matches: a|DT loud|JJ noise|NN
a|DT better|JJR Cheerios|NNPS

(\w+\|DT) (\w+\|VB[DNG])* (\w+\|N[NPS]+)+
matches: the|DT singing|VBG elephant|NN seals|NNS
an|DT apple|NN
an|DT IBM|NP computer|NN
the|DT outdated|VBD aging|VBG
Commodore|NNNP computer|NN
hardware|NN

Conclusion

- Both regular expressions and finite-state automata represent regular languages.
- The basic regular expression operations are: concatenation, union/disjunction, and Kleene closure.
- The regular expression language is a powerful pattern-matching tool.
- Any regular expression can be automatically compiled into an NFA, to a DFA, and to a unique minimum-state DFA.
- An FSA can use any set of symbols for its alphabet, including letters and words.